RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2023

FIRST YEAR [BATCH 2022-25]

Date : 26/05/2023 **ECONOMICS [HONOURS]**

Time : 11 am – 1 pm **Paper : CC4** Full Marks : 50

Answer **any five** questions : [5×6]

1. Define a Concave Function. Show that the function $f(x_1, x_2) = x_1^{1/2} x_2^{1/3}$ defined on $\mathbb{R}^2$ is strictly concave. (2+4)

2. Show that $y = x_1^{1/4} x_2^{1/3} x_3^{1/4}$ satisfies the conditions of quasiconcavity.

3. Consider the function *f* defined for all (x, y) by $f(x,y) = \frac{x^2}{2} - x + ay(x-1) - \frac{y^3}{3} + a^2 y^2$ where *a* is a constant.

   a) Prove that $(x^*, y^*) = (1-a^3, a^2)$ is a stationary point of *f*.

   b) Verify the Envelop theorem in this case. (3+3)

4. Find the optimum values of x_1 and x_2 for the function below:

   $y = 4x_1 + 2x_2 - x_1^2 - x_2^2 + x_1 x_2$. Determine whether the values correspond to global maximum or minimum. (4+2)

5. Consider the input coefficient matrix:

   $$A = \begin{bmatrix} 0.05 & 0.25 & 0.34 \\ 0.33 & 0.10 & 0.12 \\ 0.19 & 0.38 & 0 \end{bmatrix}$$

   Explain the economic meaning of 0.33 and 0. What is the economic meaning of the third column sum? (3+3)

6. Let *f* be a function on an open, convex subset U of $\mathbb{R}^n$. If x_0 is a critical point on *f* such that $Df(x_0) = 0$ and $x_0 \in U$ is found to be a global maximizer of *f* on *U*, what is the nature of *f* ?

7. Find the time path of *y* from the equation : $y_{t+1} = 2y_t - 10$. Find the equilibrium (steady state) and find whether y_t converges to the steady state or not. (3+3)

8. Suppose $K(t)$ represents the quantity of capital available at time *t*. The movement of capital is governed by the term $\dot{K} = I - \delta K$ where $\delta > 0$ is the constant rate of depreciation. Assuming *I* is constant at $\bar{I}$, find the time path of $K(t)$.

9. a) For what value of *a* is $\begin{bmatrix} a & a^2-1 & -3 \\ a+1 & 2 & a^2+4 \\ -3 & 4a & -1 \end{bmatrix}$ symmetric?

   b) Is the product of two symmetric matrices necessarily symmetric? (3+3)

10. a) Consider the matrix $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$. Find the eigen values and the eigen vector of the eigen value.

b) Find the inverse of the matrix: $\begin{pmatrix}1 & 1 & 2\\ 2 & 4 & 4\\ 3 & 3 & 7\end{pmatrix}$. (3+3)

Answer **<u>any two</u>** questions : [2×10]

11. a) Consider the utility function $U = x^{\alpha} y^{\beta}$. The consumer maximizes his utility subject to the budget constraint $M = P_x x + P_y y$. Find the Indirect Utility Function.

b) State and prove the process of deriving the Marshallian demand function from the Indirect Utility function. (6+4)

12, Let the aggregate national income of the economy Y is equal to $Y_t = C_t + I_t + G_t$ where C, I and G have their usual meanings. Let $C_t = mY_t$, $0 < m < 1$ and $I_t = \alpha\left(Y_{t-1} - Y_{t-2}\right)$. Find out the time path of Y_t assuming $G_t = \bar{G}$. Discuss briefly the possible natures of time path for Y_t.

13. Consider the utility of consuming x_1 units of good A and x_2 units of good B to be given by the utility function: $U\left(x_1, x_2\right) = \ln x_1 + \ln x_2$, and that the prices per unit of A and B are Rs. 10 and rs. 5 respectively. A consumer has at most Rs. 350 to spend on the two goods. Consider it takes 0.1 hours to consume one unit of A and 0.2 hours to consume one unit of B. A consumer has at most 8 hours to spend on consuming the two goods. How much of each good should a consumer buy in order to maximise his utility?

14. Given the demand and supply functions:

$$Q_d = 40 - 2P - 2\dot{P} - \ddot{P}$$
$$Q_s = -5 + 3P$$

with $P(0) = 12$ and $\dot{P}(0) = 1$, find $P(t)$ on the assumption that the market is always cleared. Find also the nature of the time path. (8+2)

——— × ———